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Design of discrete-time PID controller

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Ibrahim A . El- sharif

Libyan academy of Gradute Studies , Electrical and computer Eng.Department, School of Engineering
ssssharif2004@yahoo.com,

Fathi O. Hareb

Libyan academy of Gradute Studies , Electrical and computer Eng.Department, School of Engineering
harebfo@hotmail.com

Amer R. Zerek

Zawia University, Faculty of Engineering/ EE Department, Zawia, Libya,
anas_az94@yahoo.co.uk

Abstract: The discrete-time closed-loop PID controller is the most popular controller. It is more faster time response and rise time than the continuous-time PID controller. Although, we have to check the stability in a continuous-time of any digital controller, but after using z-transform to convert the system to a digital control system we use the digital signals as an input to the computer. And treat those signals inside the microprocessor by the A.T.U. after that convert them to analog to the final element.

Keywords: PID Proportional Integral Derivative ,RHS Right Hand Side ,LHS Left Hand Side ,ADC Analog to Digital Converter ,DAC Digital to Analog Converter.

1. Introduction:

In the past few decades, analog controllers have often been replaced by digital controllers whose inputs and outputs are defined at discrete time instances. The digital controllers are in the form of digital circuits, digital computers, or microprocessors. The discrete-time PID controller that means discussing the continuous signal which it has been converted from continuous-time to discrete-time , therefore always used analog to digital converter [1] .

Digital signal which has been converted it must be sampled therefore, Z-transformation and its inverse used to solve the difference equations and convert the signal at continuous-time to a signal at discrete-time . Digital to analog converter must be used after the signal treated by the discrete-time PID controller to operate the plant for example : a level control plant , the analog subsystem includes the plant as well as the amplifiers and actuators necessary to drive it. The output of the plant is periodically measured and converted to a number that can be fed back to the computer using an ADC.

2. The main structure of a digital control system :

To control a physical system or process using a digital controller, the controller must receive measurements from the system, process them, and then send control signals to the actuator that effects the control action. In almost all applications, both the plant and the actuator are analog systems. This is a situation where the controller and the controlled do not "speak the same language" and some form of translation is required. The translation from controller language (digital) to physical process language (analog) is performed by a digital-to-analog converter, or DAC. The translation from process language to digital controller language is performed by an analog-to-digital converter, or ADC. A sensor is needed to monitor the controlled variable for feedback control. The combination of the elements discussed here in a control loop is shown in 1-1 Variations on this control configuration are possible. For example, the system could have several reference inputs and controlled variables, each with a loop similar to that of Figure 1-1. The system could also include an inner loop with digital or analog control. [1]

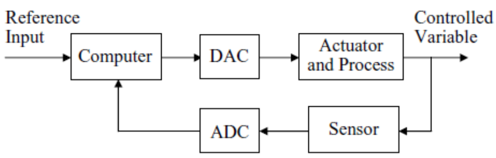

Figure 2-1 The main structure of digital control system.

Z-transform used as a key for discrete-time systems to solve the difference equations to show the output response of the control systems.

Suppose $f(t)$ is a continuous function and we sample this function at time intervals of T , thus obtaining the data

$$f(0), f(T), f(2T), \dots, F(nT), \dots$$

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let $\quad z = e^{sT} \;\; or \;\; equivalently \;\; s = \frac{1}{T}\, log(z)$

Thus:

$$F(z) = \sum_{n=0}^{\infty} f(nT) z^{-n} \qquad (2-1)$$

$$G_{ZOH}(s) = \frac{1 - e^{-sT}}{s} \qquad (2-2)$$

To convert the equation (2-2) to a z-transform :

$$G_{ZOH}(z) = \; (1 - z^{-1}) \, \mathcal{Z}\left(\frac{1}{s}\right) \qquad (2-3)$$

3. Stability and mapping between s-domain and z-domain:

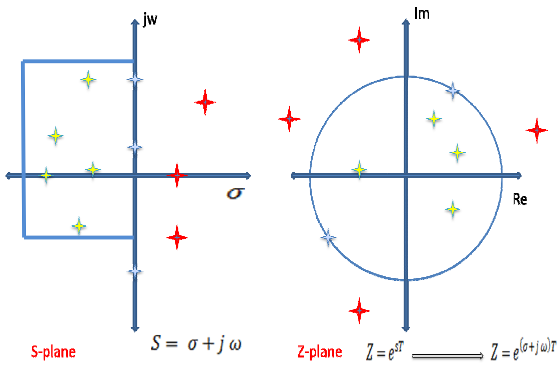

Figure 3-1 Stability between s-plane and z-plane

Clearly we could see the matching between s-domain and z-domain the circle on the R.H.S. represent the limit of stability because if the pole of z was inside the circle (the yellow stars), the system is stable. But when the pole outside the circle (the red stars) the system is unstable. If the pole is on the unit circle (the blue stars), system is critical stable. In the same way on L.H.S. the poles at left of the s-plane (the yellow stars) the system will be stable. But when the pole on the right side of the s-plane (the red stars) the system is unstable. If the pole is on the image axis of the s-plane (the blue stars), system is critical stable.

4. Root locus :

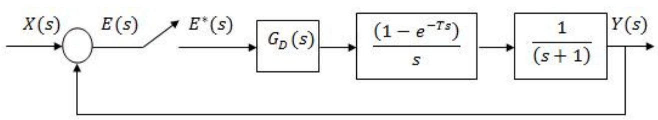

Figure 4-1 Shows the discrete-time closed-loop control system.

$$due \; to \; T = 1 \, sec \quad and \qquad G_D(z) = \frac{Kz}{(z-1)}$$

$$P(z) = \frac{Y(z)}{E(z)} = \frac{0.6321 \, Kz}{(z-1)(z - \; 0.3678)} \qquad (4-1)$$

From equation (4-1) which represents the transfer function of the discrete-time closed-loop control system shown in fig. 4-1, and we could see the both poles of the system $z = 1$, $z = 0.3678$ *and the only one zero* $z = o$

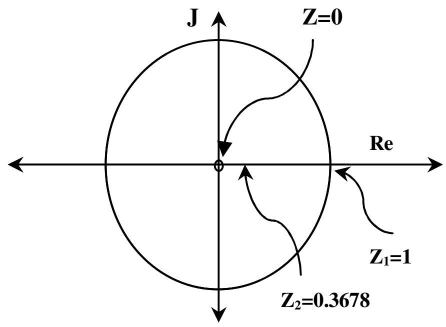

Figure 4-2 Root locus of discrete-time closed-loop system on the z-plane

By draw the root locus in z-plane we must follow steps below :

1. Leave one of the roots on R.H.S.
2. Before the next root we look back (if the number of roots were odd there are a pole in that region, otherwise -the number of poles were even- there no more poles).
3. If there is a pole –in that region- it have to move away from that place of positive real axis **Breakaway Point** to meet its zero in–negative axis- the opposite axis that point called **Break-in point**.

To find out the Break-away point and the Break-in point by :

- Take the derivative of the characteristic equation (4-2) of the system :

$$1 + P(z) = 0 \qquad (4-2)$$

$$\frac{0.6321 \, K \, z}{z^2 - \; 1.3678 \, z + 0.3678} = -1$$

$$K \; = \frac{-(z^2 - 1.3678z + 0.3678)}{0.6321Z}$$

$$= \frac{-z^2 + 1.3678z - 0.3678}{0.6321z}$$

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- **Equalize the** $\frac{dk}{dz} = 0$ **:**

$$\frac{dK}{dZ} = \frac{(0.6321Z)(-2Z+1.3678)-[(-Z^2+1.3678Z-0.3678)(0.6321)]}{(0.6321)^2\,Z^2} = 0$$

$$= \frac{-2z^2 + 1.3678z + z^2 - 1.3678z + 0.3678}{0.6321\,z^2} = 0$$

$$\frac{dK}{dz} = \frac{-z^2 + 0.3678}{0.6321\,z^2} = 0 \qquad (8-3)$$

$$z^2 = 0.3678 \quad \rightarrow \quad z = \pm\sqrt{0.3678}$$

$$z_1 = 0.606 \qquad z_2 = -0.606$$

As shown in figure 4-3 below the two open-loop poles move from their location ($z_1 = 1$, $z_2 = 0.3678$) to the Breakaway point . After that they move away from Breakaway point shaped a curve up and down until they meet each other in the Break-in point going to zero .

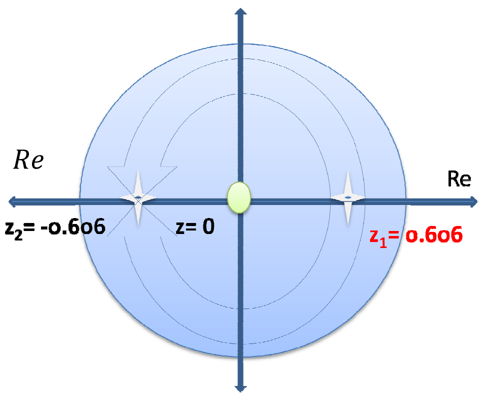

Figure 4-3 Root locus move from Breakaway point to Break-in point .

5. Design discrete-time PID controller:

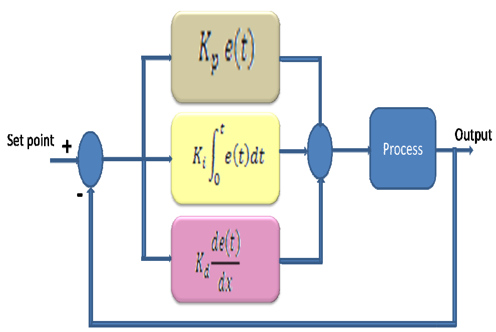

Figure 5-1 Structure of continuous PID controller .

As show in the figure 5-1 above the main component of the continuous PID controller are Proportional plus Integral plus Derivative .

Most commercial controllers provide full PID (also called three-term) control action. Including a term that is a function of the derivative of the error can ,with high-order plants, provide a stable control solution.[2]

PID controller is represented as :

$$u(t) = K_p\,e(t) + \frac{K_p}{T_i}\int_0^t e\,dt + K_p\,T_d\,\dot{e}_f(t) \qquad (5-1)$$

By taking Laplace transform :

$$U(s) = \frac{K_p\,(T_iT_d\,s^2 + T_is + 1)}{T_is}\,E(s) \qquad (5-2)$$

$$\text{Where} \quad T_i = \frac{K_p}{K_i} \quad and \quad T_d = \frac{K_d}{K_p}$$

In the same manner, digital compensators designed in the z-domain for discrete-time control system.

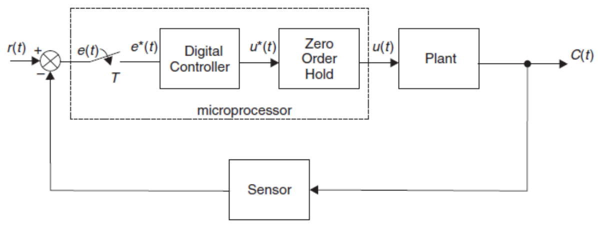

Figure 5-2 General form of a digital controller

Figure 5-2 shows the general form of digital control system. The pulse transfer function of the digital controller/compensator is written :

$$\frac{U(z)}{E(z)} = D(z) \qquad (5-3)$$

The closed-loop transfer function of the system becomes :

$$\frac{C(z)}{R(z)} = \frac{D(z)\,G(z)}{1 + D(z)\ GH(z)} \qquad (5-4)$$

The characteristic equation is :

$$1 + D(z)\ GH(z) = 0 \qquad (5-5)$$

In a continuous system, a differentiation of the error signal $e(t)$ can be represented as :

$$u(t) = \frac{de}{dt} \qquad (5-6)$$

By taking the Laplace transform with zero initial conditions :

$$\frac{U(s)}{E(s)} = s \qquad (5-7)$$

In discrete-time control system, a differentiation can be approximated to :

$$u(kt) = \frac{e(kt) - e(k-1)T}{T} \qquad (5-8)$$

The z-transform will be :

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$$\frac{U(z)}{E(z)} = \frac{1 - z^{-1}}{T} \qquad (5-9)$$

Hence, the Laplace operator can be approximated to :

$$s = \frac{1 - z^{-1}}{T} = \frac{z - 1}{Tz} \qquad (5-10)$$

Digital PID controller from the equation (5-4) , inserting equation (5-10):

$$U(z) = \frac{K_p \left(T_i T_d (\frac{z-1}{Tz})^2 + T_i(\frac{z-1}{Tz}) + 1\right)}{T_i(\frac{z-1}{Tz})} E(z) \quad (5-11)$$

By simplified to give :

$$\frac{U(z)}{E(z)} = \frac{K_p \ (b_2 z^2 + T_i b_1 z + b_0)}{z(z-1)} \qquad (5-12)$$

Where

$$b_0 = \frac{T_d}{T} \qquad (5-13)$$

$$b_1 = \left(1 - \frac{2T_d}{T}\right) \qquad (5-14)$$

$$b_2 = \left(\frac{T_d}{T} + \frac{T}{T_i} + 1\right) \qquad (5-15)$$

Tustin's Rule : also called the bilinear transformation, gives a better approximation to integration since it is based on a trapezoidal rather than a rectangular area.

$$s = \frac{2(z-1)}{T(z+1)} \qquad (5-16)$$

Substituted the value of s into the denominator of equation (5-4), still yield a digital PID controller of the form shown in equation (5-12) where :

$$b_0 = \frac{T_d}{T} \qquad (5-17)$$

$$b_1 = \left(\frac{T}{2T_i} - \frac{2T_d}{T} - 1\right) \qquad (5-18)$$

$$b_1 = \left(\frac{T}{2T_i} + \frac{T_d}{T} + 1\right) \qquad (5-19)$$

To understand the output response of the PID controller we have to see the procedures in the following Example :

Example (5-1):

The laser guided missile shown in figure 5-3 has an open-loop transfer function (combining the fin dynamics and missile dynamics) of **[2]**

$$G_2(s)\, G_3(s) = \frac{20}{s^2(s + 5)} \qquad (5-20)$$

A lead compensator, has a transfer function of :

$$G_1(s) = \frac{0.8\,(1 + s)}{(1 + 0.0625\, s)} \qquad (5-21)$$

(a) Find the z-transform of the missile by selecting a sampling frequency of at least 10 times higher than the system bandwidth .

(b) Convert the lead compensator in equation (5-21) into a digital compensator using the simple method, i.e. equation (5-10) and find the step response of the system.

(c) Convert the lead compensator in equation (5-21) into a digital compensator to find the step response of the system, thus use Tustin's rule, to find the step response of the system.

(d) Compare the response found with the continuous step response, and convert the compensator that is closest to its difference equation.

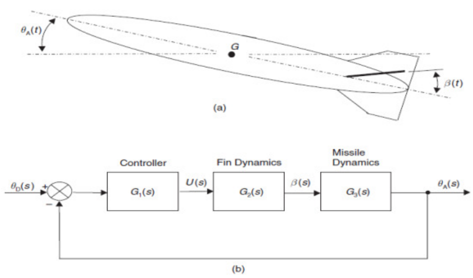

Figure 5-3 Laser guided missile

Solution :

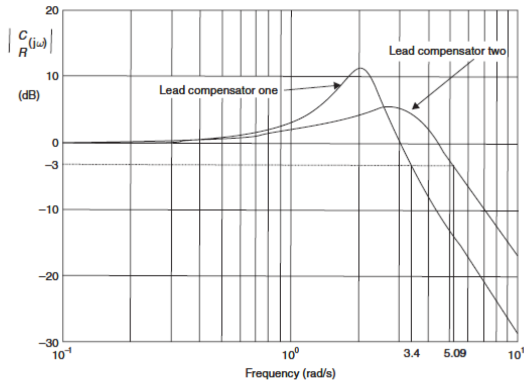

Figure 5-4 Closed-loop frequency response for both lead compensator designs.

Table 5-1 System frequency domain performance :

Closed-loop peak M_p	Gain margin	Bandwidth	Phase margin
5.5 dB	13.75 dB	5.09 rad/s	30.6°

(a) From figure 5-4 lead compensator two, the bandwidth is 5.09 rad/s or 0.81 Hz . Ten times this is 8.1 Hz, so select a sampling frequency of 10Hz, i.e. =0.1 seconds. For a sample and hold device cascaded with the missile dynamics.[2]

$$G(s) = \left(\frac{1 - e^{-Ts}}{s}\right)\left[\frac{20}{s^2(s + 5)}\right] \qquad (5-22)$$

$$G(s) = (1 - e^{-Ts})\left[\frac{20}{s^3(s + 5)}\right] \qquad (5-23)$$

For $T = 0.1\ s$, *equation* $(4-26)$ *will be as below* :

$$G(z) = \frac{0.00296\,z^2 + 0.01048\,z + 0.0023}{z^3 - 2.6065\,z^2 - 0.6065} \qquad (5-24)$$

(b) Substituting the value of s which is in equation (5-10) into the lead compensator given in equation (5-21) :

$$D(z) = 0.8\left[\frac{\frac{Tz + (z-1)}{Tz}}{\frac{Tz + 0.0625\,(z-1)}{Tz}}\right]$$

$$D(z) = \frac{5.4152\,z - 4.923}{z - 0.3846} \qquad (5-25)$$

(c) Using Tustin's rule by substituting the value of s of equation (5-16) into lead compensator equation (9-23):

$$D(z) = 0.8\left[\frac{\frac{T(z+1) + 2(z-1)}{T(z+1)}}{\frac{T(z+1) + 0.0625\,[2(z-1)]}{T(z+1)}}\right]$$

$$D(z) = \frac{U(z)}{E(z)} = \frac{7.467\,z - 6.756}{z - 0.111} \qquad (5-26)$$

(d) From Figure (5-5) below, we could see clearly that the digital compensator using Tustin's rule is closest to the continuous response. From equation (5-26)

$$D(z) = \frac{U(z)}{E(z)} = \frac{7.467 - 6.756z^{-1}}{1 - 0.111z^{-1}} \qquad (5-27)$$

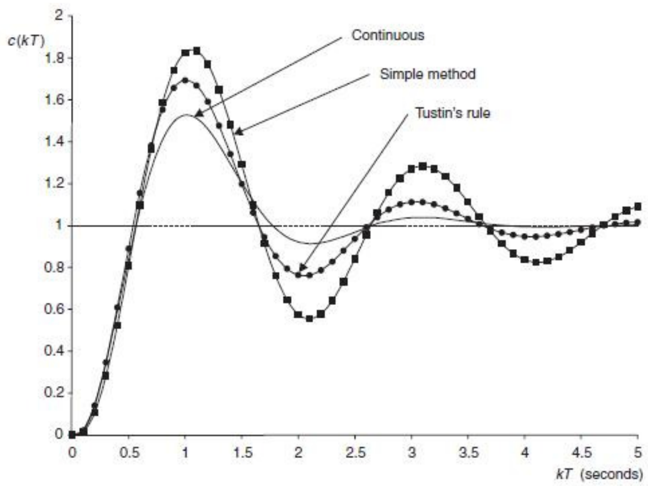

Figure 5-5 Comparison between discrete digital compensator in both (simple method, Tustin's rule) response and continuous response.

The difference equation for the digital compensator is :

$$u(kt) = 0.111\,u(k-1)T + 7.467\,e(kt) - 6.756\,e(k-1)T \quad (5-28)$$

6. Result discussion of the comparison between an analog and digital PID controller :

For a special comparison of The time response between an analog PID controller and digital PID controller shown in figure 6-1 after finding the Laplace transformation , tune the parameters of the PID controller and convert the transfer function to the z-transformation , and plot the analog PID controller with gain $K = 100$ and both digital PID controller gains but with K= 9.62,13.2, the system has $\zeta = 0.7$, 0.5, and $\omega_n = 43.2, 46.2\,rad/s$, respectively. Both designs have a sufficiently fast time constant, but the second damping ratio is less than the specified value of 0.7. Lower gains give an unacceptably slow analog design. The time response for the high-gain digital design is very fast. However, it has an overshoot of over 4% but has a settling time of 5.63 s.

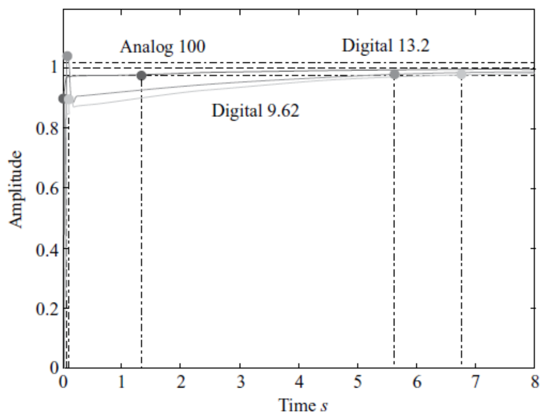

Figure 6-1 Time step response for the digital PID design with K = 9.62 (light gray), K = 13.2 (dark gray) and for analog design (black) K =100.

The digital design for ζ = 0.7 has a much slower time response than its analog counterpart . It is possible to improve the design by trial and error, including redesign of the analog controller, but the design with ζ = 0.5 may be acceptable. One must weigh the cost of redesign against that of relaxing the design specifications for the particular application at hand. The final design must be a compromise between speed of response and relative stability.[1]

7. Laboratory Experiments

```
>> num=[20];
>> den=conv([1 0 0],[1 5]);
>> Ts=0.1;
>> ncomp=0.8*[1 1];
>> dcomp=[0.0625 1];
```

```
>> [nol,dol]=series(ncomp,dcomp,num,den);
>> [ncl,dcl]=cloop(nol,dol);
>> [numd,dend]=c2dm(num,den,Ts,'zoh');
>> [ncomd,dcomd]=c2dm(ncomp,dcomp,Ts,'tustin');
>> printsys(num,den,'s')
 num/den =
        20
   -----------
   s^3 + 5 s^2
>> printsys(num,den,'z')
 num/den =
        20
   -----------
   z^3 + 5 z^2
>> printsys(ncomp,dcomp,'s')
 num/den =
   0.8 s + 0.8
   ------------
   0.0625 s + 1
>> printsys(ncomp,dcomp,'z')
 num/den =
   0.8 z + 0.8
   ------------
   0.0625 z + 1
>> [nold,dold]=series(ncomd,dcomd,numd,dend);
>> [nold,dold]=cloop(nold,dold);
>> [nold,dold]=series(ncomd,dcomd,numd,dend);
>> [ncld,dcld]=cloop(nold,dold);
>> subplot(211),step(ncl,dcl);
>> subplot(212),dstep(ncld,dcld);
```

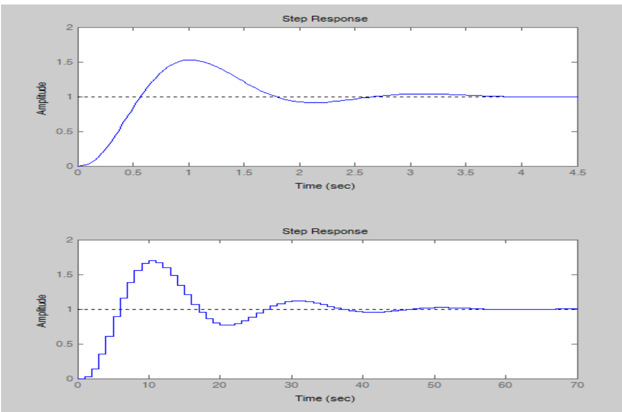

Figure 6-2 Laboratory experiment : Comparison between analog and digital PID controller

Conclusion

Due to both figures (4-5 and 4-6) we can clearly see the difference between the continuous closed-loop PID controller and digital closed-loop PID controller in the output responses even if they are have the same parameters like $\zeta = 0.7$.
There all a lot of benefits when we use digital PID controllers instead of using continuous PID controller they could summarized in following points :

1. **Accuracy.** Digital signals are represented in terms of zeros and ones to represent a single number. This involves a very small error as compared to analog signals where noise and power supply drift are always present.
2. **Implementation errors.** The errors that result from digital representation and arithmetic are negligible. By contrast, the processing of analog signals is performed using components such as resistors and capacitors with actual values that vary significantly from the nominal design values.
3. **Flexibility.** An analog controller is difficult to modify or redesign once implemented in hardware. A digital controller is implemented in firmware or software, and its modification is possible without a complete replacement of the original controller.
4. **Speed.** The speed of computer hardware has increased exponentially. This increase in processing speed has made it possible to sample and process control signals at very high speeds.
5. **Cost.** Although the prices of most goods and services have steadily increased, the cost of digital circuitry continues to decrease.

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2. Roland S. Burns, ' Advanced Control Engineering ' , Butterworth-Heinemann,2001 .